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PROBLEMS WITH DISCONTINUOUS BOUNDARY CONDITIONS DESCRIBING LAMINAR FLOWS AT HIGH REYNOLDS NUMBERS†

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The asymptotic structure of perturbed laminar flows, which are described by problems with unsteady, discontinuous boundary conditions, is investigated. As a result of an asymptotic analysis, the structure of the flow domains is found and appropriate boundary conditions are formulated. Numerical solutions are obtained for one of the processes, which is described by an inhomogeneous Burgers equation and corresponds to discontinuities in the vertical longitudinal velocities, as well as to the blow out of a tangential jet.

In boundary-layer theory, discontinuous boundary conditions are encountered in problems which describe a flow close to the edges of profiles or wings, close to lines of discontinuity in the surfaces around which the flow occurs, etc. For example, the transition from a flow in a boundary layer on a plate of zero thickness at zero angle of attack to the flow in the wake is associated with a discontinuity in the normal derivative of the longitudinal velocity, which is proportional to the friction on the surface of the plate and is equal to zero on the axis of the wake. An investigation of flows of this type, based on the boundary-layer equations, has shown that the vertical velocity at the outer edge of the wake increases without limit as the rear edge of the plate is approached. The assumptions of boundary-layer theory therefore no longer hold.

It has been shown [1, 2], by an asymptotic analysis of the Navier–Stokes equations, that the increase in the vertical velocity at the outer edge of a viscous flow due to a change in the displacement thickness of the wake is bounded and does not exceed the values at which the pressure perturbation induced in the outer inviscid flow begins to affect the change in the displacement thickness. Similar effects of a local, strong viscous-inviscid interaction have also been discovered in the neighbourhood of the point of separation of a boundary layer from a smooth surface in supersonic flow [3, 4]. Subsequent analysis [5] showed that a complex flow structure arises close to the rear edge of a plate and that this structure contains a number of embedded domains, in which the flow is described by the complete system of Navier–Stokes equations, by a system of boundary layer equations with an induced pressure gradient and other equations.

Other types of discontinuity, e.g. in the surface velocity [6], in the derivative of the stream function (distributed blowing) [7], and in the surface temperature [8, 9], are also characterized by a complex structure of the perturbed domain of the flow.

The unsteady perturbations generated by discontinuous boundary conditions have been investigated to a significantly lesser extent, although the study of such processes is necessary in order to determine the characteristics of the non-linear stability of laminar flows at high Reynolds numbers.

1. STRUCTURE OF THE PERTURBED FLOW DOMAINS

In order to construct a diagram of the domains of perturbed flow, we will consider, as an example, the problem of flow close to a domain of discontinuity in the velocity on a plate which has a moving segment at a distance l from its leading edge. The velocity of this segment is equal to u_w.

The following notation is adopted for the Cartesian coordinates, measured along the surface around which the flow occurs and along a normal to it, the components of the velocity vector, the density, pressure, coefficient of viscosity, and total enthalpy: xl, yl, tl/u_∞, uu_∞, υu_∞, $\rho\rho_\infty$, $p\rho_\infty u_\infty^2$, $\mu\mu_\infty$, Hu_∞^2. Dimensional functions in the free stream are given the subscript ∞.

We will first consider the structure of the unperturbed steady flow.

The discontinuity in the velocity of the streamlines passing close to the surface with a velocity u_w and the streamlines with almost zero velocities can lead to the formation of a new boundary layer downstream

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from the point of discontinuity. The conditions, under which a new boundary layer is developed as a result of the effect of discontinuous boundary conditions, are obtained below. The structure of the local perturbed flow depends on the magnitude of the local Reynolds number (which is constructed using the local parameters).

An estimate of the thickness of the boundary-layer developed follows from the condition that the orders of magnitude of the terms describing the effect of inertial forces and viscosity in the longitudinal momentum equation

$$y \sim \varepsilon u_w^{-1/2} x^{1/2}, \quad \varepsilon = \mathrm{Re}^{-1/2}, \quad \mathrm{Re} = \rho_\infty u_\infty l / \mu_\infty \tag{1.1}$$

are equal.

For a fixed magnitude of the surface velocity and a varying thickness of the boundary-layer developed, the friction in it decreases monotonically as the longitudinal coordinate increases. Using estimate (1.1), it is possible to find the distance x_1 at which the friction in the boundary layer developed becomes comparable with the friction in the main boundary layer.

$$u_w/\varepsilon \sim 1/\varepsilon, \quad x_1 \sim u_w^3 \tag{1.2}$$

The relationships obtained are conveniently represented graphically in the form of graphs of $\ln x/\ln \varepsilon = f(\ln u_w/\ln \varepsilon)$. Relation (1.2) is then represented by the line OB (Fig. 1).

From estimate (1.1), it is also possible to find the distance from the point of discontinuity in the boundary condition at which the thickness and length of the domain of non-linear perturbations become of the same orders of magnitude

$$x_2 \sim \varepsilon^2/u_w \tag{1.3}$$

where, in fact, the propositions of boundary-layer theory no longer hold.

Relation (1.3) is represented by the line AB in Fig. 1.

The fact that the longitudinal and transverse scales are identical then also leads to equality in the orders of magnitude of the perturbed longitudinal and transverse velocities. Since relation (1.1) was obtained under the assumption that the inertial and viscous forces are of the same order of magnitude, it can be shown that the flow in a domain with a scale $x_2 \sim y_2 \sim \varepsilon^2/u_w$ is described by the complete system of Navier–Stokes equations for an incompressible fluid. The above-mentioned domain also arises in the flow analysis near the leading edge of a plate of zero thickness.

We will now estimate the effect of the boundary layer which is developed due to the discontinuity in the boundary conditions on the flow in the initial boundary layer. Physically, this effect is expressed in the absorption of gas from the main boundary layer. An estimate of the magnitude of the vertical velocity in the boundary layer developed follows from (1.1) and consideration of the continuity equation and has the form

$$v \sim \varepsilon u_w^{1/2} x^{-1/2}, \quad \psi \sim v\, x \sim \varepsilon u_w^{1/2} x^{1/2}$$

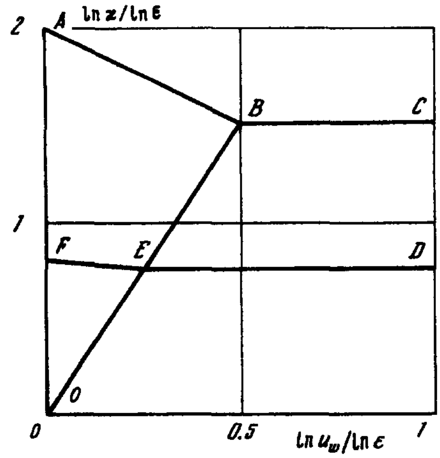

Fig. 1.

The absorption of such a flow at a distance x from the initial boundary layer leads to a change in its thickness. In order to determine the magnitude of this change, we make use of a representation of the velocity profile in the main boundary layer at small distances compared with the boundary-layer thickness $u \sim y/\varepsilon$. Correspondingly, the gas flow across the main boundary layer at a distance y from the surface is estimate in the following manner: $\psi \sim y^2/\varepsilon$. Consequently, the estimate for the change in the displacement thickness of the main boundary layer has the form

$$\Delta\delta \sim \varepsilon^{1/2}\psi^{1/2} \sim \varepsilon u_w^{1/4} x^{1/4}$$

This change in the displacement thickness induces a corresponding pressure perturbation in the outer inviscid flow

$$\Delta p \sim \Delta\delta / x \sim \varepsilon u_w^{1/4} x^{-3/4}$$

This estimate follows from the linear theory of inviscid (subsonic or supersonic) flows. The use of this theory is justified if the distance x_3, at which the above mentioned effects are substantial, exceeds the thickness of the main boundary layer $\delta \sim \varepsilon$. The conditions under which the assumption that $x_3 > O(\varepsilon)$ holds will be determined below.

The estimate for the pressure perturbation enables one to find the distance x_3 at which the induced pressure gradient exerts a non-linear action in the domain of the main boundary layer close to the wall. It is assumed that the gas flow is actually absorbed from this domain and that, in fact, the change in the thickness of this domain determines the overall change in the displacement thickness of the boundary layer

$$\Delta p \sim u_3^2, \quad x_3 \sim \varepsilon^{4/5} u_w^{-1/5} \tag{1.4}$$

From relations (1.4), the total change in the displacement thickness is represented by line EF in Fig. 1. Using estimate (1.4), it is possible to write down the condition for which the length of the interaction domain exceeds the boundary-layer thickness.

$$u_w < 1/\varepsilon$$

which is necessarily assumed to be accomplished.

The characteristic points B and E lie at the intersection of the line OB with the lines AB and FE. For flow conditions corresponding to the points B and E, the orders of magnitude of the friction in the perturbed domain and in the main boundary layer are characteristically the same. Point B then corresponds to flow which is described by a system of Navier–Stokes equations while flow which is described by a system of equations from the theory of "free interaction" corresponds to point E.

Estimate (1.3) does not hold for the range of variation of the parameter u_w which is located to the right of point B or in cases when linear perturbation conditions are realized due to the large relative effect of viscous forces. The equality of the orders of magnitude of the terms describing the effect of viscous and inertial forces leads to the estimate

$$x_4 \sim \varepsilon^{3/2} \tag{1.5}$$

Relation (1.5) is described by the line BC in Fig. 1. Similar reasoning can be used to find the distance x_5 at which linear viscous-inviscid interaction processes occur. Relation (1.4) does not hold over the range of variation of the parameter u_w which is located to the right of point E. The estimate

$$x_5 \sim \varepsilon^{3/4} \tag{1.6}$$

follows for the distance x_5. The line ED corresponds to this.

The diagram of the domains of perturbed flow shown in Fig. 1 enables one to determine the sizes of these domains and the nature of the flow in them for a given amplitude of the parameter u_w. Thus, the action of a perturbation with an amplitude $O(\varepsilon^{1/4}) \leq u_w \leq O(1)$ leads to the appearance, close to the discontinuity, of a domain with dimensions which are determined by the line AB, the flow in which is described by a system of Navier–Stokes equations for an incompressible fluid. The next domain, which is larger in size, is a domain with a longitudinal dimension which is determined by the line EF where the flow is described in the first approximation by Burgers equation. In this case, a compensating interaction process [10] is realized at intermediate distances when the parameters in the domain of non-linear perturbations are varied and the effect of viscosity is unimportant. The absence of viscous terms in the equations describing the perturbed flow requires the introduction of subdomains in which the effect of viscosity is of the same order of magnitude as the effect of inertial forces. At the same time, a domain exists with a length determined by the line OB, in which the effect of viscosity is substantial and in which the surface friction is of the same order of magnitude as the friction in the boundary layer. As noted above, point E corresponds to the general case when non-linear processes of the equalization of the friction occur in a single domain, that is the domain of "free interaction" [3–4]. When the parameter u_w is varied within the range $O(\varepsilon^{1/4}) \leqslant u_w \leqslant O(\varepsilon^{1/2})$, yet another domain, with a length which is determined by the line BE, appears, in addition to the domain which has been mentioned above, where the flow is described by a system of Navier–Stokes equations. Here, the flow is described by a system of boundary-layer equations with a compensating interaction condition. Levelling of the magnitude of the surface friction occurs in this domain. Finally, when $u_w \sim \varepsilon^{1/2}$, equalization of the friction occurs immediately in this domain, where the flow is described by a Navier–Stokes equations.

Hence, a system of embedded domains with different scales in the longitudinal direction arise close to the point (line) of discontinuity. The size of each of these domains for a fixed value of u_w can be determined using the diagram in Fig. 1. Account must also be taken of the fact that each domain, with a longitudinal dimension which is greater in order of magnitude than the thickness of the boundary layer, consists of subdomains of different transverse size.

The characteristic times for all domains of the perturbed flow can be determined by using the diagram in Fig. 1 and the estimates which have been presented above. These characteristic values of the time are equal to the ratio of the lengths of the domains to the characteristic velocity values in these domains. Consequently, in the case of the domain under consideration, which consists of a system of embedded subdomains, the greatest characteristic time will correspond to the subdomain with the smallest characteristic longitudinal velocity and quasi-steady-state processes in the remaining subdomains will correspond to unsteady processes in the subdomain with the greatest time. It follows from the above estimates that the smallest longitudinal velocity is characteristic for the domain in which non-linear changes occur.

Account also has to be taken of the fact that different characteristic times will also correspond to domains with different longitudinal dimensions. For instance, in the case of the domains corresponding to the lines in Fig. 1, we have the estimates: $t_2 \sim \varepsilon^2 u_w^{-2}$ on AB, $t_3 \sim \varepsilon^{3/5} u_w^{-2/5}$ on EF and $t_1 \sim u_w^2$ on OB.

For our further analysis, we note that the shortest time, when $u_w = o(\varepsilon^{1/2})$, is characteristic for the flow domain corresponding to the line AB, the next is the time for the domain EF, which follows with respect to its size, and the characteristic time for the domain corresponding to the line OE turns out to be the greatest.

2. ANALYSIS OF NON-LINEAR PROCESSES

It is important in the subsequent analysis that, in the principal approximation, the perturbed flow in a domain characterized by a smaller longitudinal scale has no effect on the flow in a domain with a larger longitudinal scale. The flow close to the leading edge of a plate of zero thickness, arranged at zero angle of attack to the free stream, serves as an analogous example.

We will confine ourselves to flow conditions which correspond to the domain EF in Fig. 1. As has been shown above, the flow in domain 3 is found to be inviscid and the discontinuity in the boundary conditions leads to the appearance of non-linear perturbations. In addition to the above mentioned hierarchy of rates in the longitudinal direction, subdomains are associated with each domain (the length of which exceeds the thickness of the boundary layer) and the transverse dimensions of these subdomains are determined respectively by the thickness of the boundary layer which is formed, by the thickness of the domain with non-linear changes, by the thickness of the main boundary layer and the longitudinal size of the domain of perturbed flow close to the discontinuity (Fig. 2).

We will now consider the flow in the domain where non-linear changes in the longitudinal velocity occur, that is, domain 3, for which the following representations of the stream functions and coordinates are characteristic

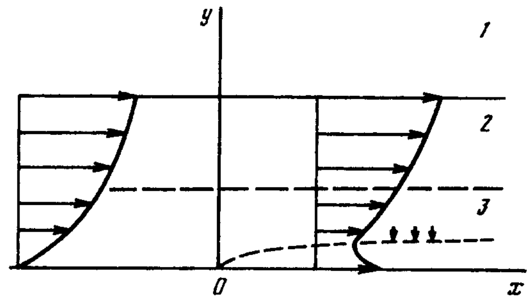

Fig. 2.

$$
\begin{aligned}
&x = 1 + x_3\varepsilon^{4/5}u_w^{-1/5}, \quad y = y_3\varepsilon^{6/5}u_w^{1/5}, \quad t = t_3\varepsilon^{3/5}u_w^{-2/5} \\
&(u,v) = (\varepsilon^{1/5}u_w^{1/5}u_3, \ \varepsilon^{3/5}u_w^{3/5}v_3) + \ldots \\
&(p,\rho) = \left(\frac{1}{\gamma M_\infty^2} + \varepsilon^{2/5}u_w^{2/5}p_3, \rho_w\right) + \ldots
\end{aligned} \tag{2.1}
$$

Substituting expansion (2.1) into the system of Navier–Stokes equations, we obtain, on taking the limit

$$\varepsilon \to 0, \quad u_w \to 0, \quad \varepsilon^{1/4}/u_w \to 0 \tag{2.2}$$

the following system

$$
\begin{aligned}
&\frac{\partial u_3}{\partial t_3} + u_3\frac{\partial u_3}{\partial x_3} + v_3\frac{\partial u_3}{\partial y_3} + \frac{1}{\rho_w}\frac{\partial p_3}{\partial x_3} = 0 \\
&\frac{\partial u_3}{\partial x_3} + \frac{\partial v_3}{\partial y_3} = 0, \quad \frac{\partial p_3}{\partial y_3} = 0
\end{aligned} \tag{2.3}
$$

The boundary conditions where $x_3 \to -\infty$ are determined by the solution for the domain close to the wall in the main boundary layer

$$u_3 = ay_3, \quad v_3 = 0, \quad p_3 = 0 \tag{2.4}$$

The following representations are characteristic in the case of domain 2, which is located above domain 3

$$x = 1 + \varepsilon^{4/5}u_w^{-1/5}x_2, \quad y = \varepsilon y_2, \quad t = \varepsilon^{3/5}u_w^{-2/5}t_2 \tag{2.5}$$

$$
\begin{aligned}
&u(x,y,t,\varepsilon,u_w) = u_0(y_2) + \varepsilon^{1/5}u_w^{1/5}u_2(x_2,y_2,t_2) + \ldots \\
&v(x,y,t,\varepsilon,u_w) + \varepsilon^{2/5}u_w^{2/5}v_2(x_2,y_2,t_2) + \ldots \\
&p(x,y,t,\varepsilon,u_w) = \frac{1}{\gamma M_\infty^2} + \varepsilon^{2/5}u_w^{2/5}p_2(x_2,y_2,t_2) + \ldots \\
&\rho(x,y,t,\varepsilon,u_w) = \rho_0(y_2) + \varepsilon^{2/5}u_w^{2/5}\rho_2(x_2,y_2,t_2) + \ldots
\end{aligned} \tag{2.6}
$$

Substituting expansions (2.6) into the system of Navier–Stokes equations and taking the limit (2.2) we obtain the system

$$u_0\frac{\partial u_2}{\partial x_2} + v_2\frac{\partial u_2}{\partial y_2} = 0, \quad \frac{\partial u_2}{\partial x_2} + \frac{\partial v_2}{\partial y_2} = 0 \tag{2.7}$$

The solution of (2.7) has the form

$$u_2 = A(x_2, t_2)\frac{du_0}{dy_2}, \quad v_2 = -u_0\frac{\partial A}{\partial x_2} \tag{2.8}$$

The following representations of the functions and coordinates can be introduced in domain 1, which contains the streamlines of the inviscid outer flow

$$x = 1 + \varepsilon^{4/5}u_w^{-1/5}x_1, \quad y = \varepsilon^{4/5}u_w^{-1/5}y_1, \quad t = \varepsilon^{3/5}u_w^{-2/5}t_1$$
$$u(x, y, t, \varepsilon, u_w) = 1 + \varepsilon^{2/5}u_w^{2/5}u_1(x_1, y_1, t_1) + \dots \tag{2.9}$$
$$\rho(x, y, t, \varepsilon, u_w) = 1 + \varepsilon^{2/5}u_w^{2/5}\rho_1(x_1, y_1, t_1) + \dots$$

The representations for the remaining functions, apart from the replacement of a subscript, are the same as those in (2.6).

Substituting expansions (2.9) into the system of Navier–Stokes equations and taking the limit (2.2) we obtain

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial p_1}{\partial x_1} = 0, \quad \frac{\partial v_1}{\partial x_1} + \frac{\partial p_1}{\partial y_1} = 0, \quad \frac{\partial \rho_1}{\partial x_1} + \frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} = 0, \quad p_1 = \frac{\rho_1}{M_\infty^2} \tag{2.10}$$

whence it is possible to obtain the well-known wave equation (when $M_\infty > 1$) of the linear theory of supersonic flows, the solution of which is Ackeret's formula [11]

$$\sqrt{M_\infty^2 - 1}\, p_1(x_1, 0, t_1) = v_1(x_1, 0, t_1) \tag{2.11}$$

In the case of subsonic flows, the solution of system of equations (2.10) has the form

$$\frac{\partial p_1(x_1, 0, t_1)}{\partial x_1} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{(s - x_1)}\frac{\partial v(s, 0, t_1)}{\partial s}ds \tag{2.12}$$

Matching the solutions in domains 1 and 2, we find

$$p_1(x_1, 0, t_1) = \begin{cases} -\dfrac{1}{\sqrt{M_\infty^2 - 1}}\dfrac{\partial A}{\partial x_1}, & M_\infty > 1 \\ -\dfrac{1}{\pi}\displaystyle\int_{-\infty}^{\infty}\frac{1}{(s - x_1)}\frac{\partial A}{\partial s}ds, & M_\infty < 1 \end{cases} \tag{2.13}$$

Matching the solutions in domains 2 and 3 we obtain

$$aA = u_3(x_3, y_3, t_3) - ay_3, \quad y_3 \to \infty \tag{2.14}$$

The solution for domain 3 can be sought in the form

$$u_3(x_3, y_3, t_3) = ay_3 + aA(x_3, t_3)$$

for which the system of equations (2.3) takes the form

$$a\frac{\partial A}{\partial t_3} + a^2 A\frac{\partial A}{\partial x_3} + \frac{1}{\rho_w}\frac{\partial p_3}{\partial x_3} + av_{3w} = 0 \tag{2.15}$$

and the pressure perturbation is given by formula (2.13), since $p_3(x_3, t_3) = p_1(x_1, 0, t_1)$.

Hence, the flow in the domain of non-linear perturbations in the neighbourhood of the line of discontinuity is described by inhomogeneous Benjamin–Ohno equations (in the case of a subsonic external flow) or Burgers equations (in the case of a supersonic external flow).

3. EXAMPLES

We will now present the results of the numerical solution of Burgers equation.

In the examples considered below, the perturbation introduced by the discontinuity in the boundary conditions affects the flow in domain 3 through the vertical velocity. This rate of suction (blowing) is determined by the solution for the boundary layer developed. The boundary conditions for the boundary-layer equations are determined by the form of the discontinuity. It is important that a single formulation of problem (2.15) corresponds, apart from an algebraic replacement of the variables and with different velocity distributions υ_{3w}, to all the forms of discontinuities in the boundary conditions considered below. It should also be noted that, with the exception of the first of the examples considered, the flow in the boundary layer developed is characterized by asymptotically large values of the longitudinal velocity and, correspondingly, by smaller values of the characteristic time than in domain 3. Hence, assuming the unsteady nature of the flow in domain 3, we arrive at quasi-steady-state processes in the boundary layer developed.

Blowing with a velocity υ_w starting at the instant of time $t_3 = 0$ in the domain of $x_3 > 0$, where the porous segment of the surface is located. It has been shown in [7] that blowing generates a flow in domain 3 which is described by system of equations (2.2) in the case when $O(\varepsilon^{3/4}) < \upsilon_w < O(1)$. A steady-state solution of the problem has also been obtained. The evolution of the transition to this steady state is studied below. In this case, other asymptotic expressions are characteristic for domain 3

$$x = 1 + (\varepsilon^{-3} a^3 \beta \rho_w \upsilon_w)^{-1/3} X, \quad t = (\varepsilon^{-3} a^3 \beta \rho_w \upsilon_w^2)^{-1/3} T$$
$$p = \frac{1}{\gamma M_\infty^2} + (\beta^{-2} \rho_w \upsilon_w^2)^{1/3} P, \quad P = -\frac{\partial A}{\partial X} \tag{3.1}$$

Substituting expression (3.1) into system of equations (2.15), we obtain

$$\frac{\partial A}{\partial T} + A\frac{\partial A}{\partial X} - \frac{\partial^2 A}{\partial X^2} + F(X,T) = 0 \tag{3.2}$$

$$A(X,0) = 0, \quad A(-\infty,T) = 0, \quad \frac{\partial A(\infty,T)}{\partial X} = 0$$
$$F(X,T) = \begin{cases} 0, & X < 0, \ T \geqslant 0 \\ 1, & X \geqslant 0, \ T > 0 \end{cases} \tag{3.3}$$

It should be noted that Burgers equation also describes other flow conditions in a boundary layer which are not necessarily generated by a discontinuity in the boundary conditions. Examples of such processes have been investigated previously [12, 13].

The solution of problem (3.2), (3.3) was obtained numerically using the method of finite differences. The function $A(X, T)$ at different instants of time is shown in Fig. 3. For large time values, the numerical solution of the unsteady problem is identical to the solution of the steady-state problem obtained earlier in [7].

Motion of the surface starting at the time $t_3 = 0$ in the domain $x_3 > 0$ with a velocity u_w. The influence of this form of discontinuity also affects the flow in domain 3 through the occurrence of a suction velocity, the value of which is determined by the ejection properties of the boundary layer developed. The flow in the boundary layer is described by the Blasius equation

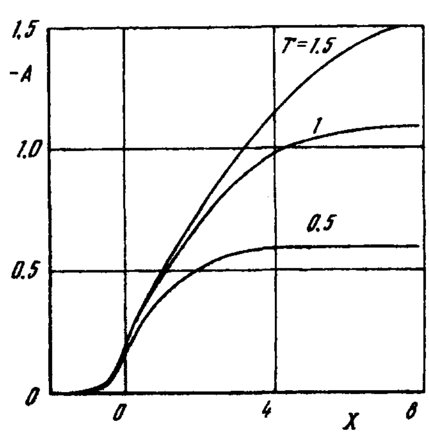

Fig. 3.

$$f''' + ff'' = 0$$

$$\psi = (2\varepsilon^2 u_w \rho_w^{-1} \mu_w x)^{1/2} f(\eta), \quad \eta = (2\varepsilon^{-2} u_w \rho_w \mu_w^{-1} x^{-1})^{1/2} y$$

with the boundary conditions

$$f(0) = 0, \; f'(0) = 1, \; f'(\infty) = 0$$

corresponding to the problem being considered.

The solution required for the subsequent analysis has the form

$$f(\infty) = C_0, \quad C_0 \approx 1.23$$

Then

$$v_{3w} = -(2^{-1} \rho_w^{-1} \mu_w C_0^2 x_3^{-1})^{1/2}$$

The change of variables

$$x = 1 + (2\varepsilon^4 u_w^{-1} a^{-6} \beta^{-4} \rho_w^{-3} \mu_w^{-1} C_0^{-2})^{1/5} X$$

$$t = (4\varepsilon^3 u_w^{-2} a^{-7} \beta^{-3} \rho_w^{-1} \mu_w^{-2} C_0^{-4})^{1/5} X$$

$$p = (\gamma M_\infty^2)^{-1} + (4^{-1} \varepsilon^2 u_w^2 a^2 \beta^{-2} \rho_w \mu_w^2 C_0^{-4})^{1/5} P, \quad P = -\partial A / \partial X$$

reduces the equation to the form of (3.2), where

$$F(X,T) = \begin{cases} 0, & X < 0, \; T \geq 0 \\ -X^{-1/2}, & X \geq 0, \; T > 0 \end{cases} \tag{3.4}$$

with the boundary conditions

$$A(X,0) = 0, \quad A(-\infty,T) = 0, \quad \frac{\partial A(\infty,T)}{\partial X} = 0 \tag{3.5}$$

The results of the numerical solution of problem (3.2), (3.4), (3.5) are shown in Fig. 4, where the function $A(X, T)$ is presented at different instants of time. As time passes, the solution describing the perturbed flow upstream from the discontinuity reaches the solution of the steady-state problem quite rapidly. The solution describing the perturbed flow downstream from the discontinuity is characterized by the motion of a solitary wave. This is seen more clearly in Fig. 5 where the distributions of the pressure perturbation at different instants of time are shown.

Tangential blowing starting at the time $t_3 = 0$ through a slot in the surface when $x_3 = 0$. It is assumed that a flow is formed close to the surface of the body and that this flow is described by the well-known self-similar solution [14]

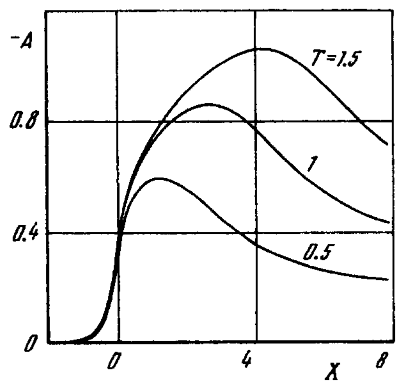

Fig. 4.

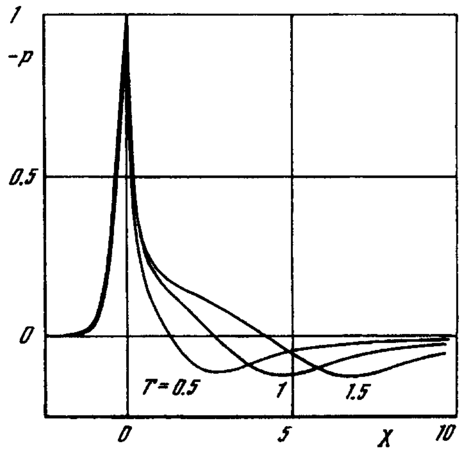

Fig. 5.

for a wall jet which is blown into a submerged space. The steady-state solution of this problem has been obtained in [15] and it has been shown that, in the case of a change in the invariant

$$I = \int_0^\infty u^2 d\lambda \int_0^\lambda u dy$$

within the range $O(\varepsilon^{13/4}) < I < O(1)$ in domain 3, a flow occurs which is described by Burgers equation (3.2). As in the preceding case, the effect of the wall jet on the flow in domain 3 is brought about by the injection velocity at the outer boundary of the wall jet. The expansions

$$x = 1 + (\varepsilon^{10} a^{-12} \beta^{-8} \rho_w^{-7} \mu_w^{-1} I^{-1} C_1^{-4})^{1/9} X$$

$$t = (\varepsilon^2 a^{-15} \beta^{-7} \rho_w^{-5} I^{-2} C_1^{-8})^{1/9} T$$

$$p = (\gamma M_\infty^2)^{-1} + (\varepsilon^{-2} a^6 \beta^{-2} \rho_w^5 \mu_w^2 I^2 C_1^8)^{1/9} P$$

$$v_w = 4^{-1} (\varepsilon^2 \rho_w^{-1} \mu_w I C_1^4 x^{-3})^{1/4}, \quad C_1 \approx 2.515$$

hold in the case of the problem under consideration.

The equation for the function $A(X, T)$ has the form of (3.2) where

$$F(X,T) = \begin{cases} 0, & X < 0, \quad T \geqslant 0 \\ -X^{-3/4}, & X \geqslant 0, \quad T > 0 \end{cases} \tag{3.6}$$

The results of the numerical solution of Eq. (3.2), subject to conditions (3.6) and (3.4), are qualitatively identical. As in the preceding case, the blowing out of a jet leads to the formation of a solitary wave which moves downstream.

All of the examples which have been considered above are characterized by the formation of a domain of increased pressure which moves downstream. However, this does not lead to the separation of the boundary layer since, according to the assumption, the flow in it is independent in the first approximation of the flow in the domain of non-linear perturbations.

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